Corona COVID-19 spread - a nonlinear modeling and simulation☆

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ABSTRACT

This paper presents a non-linear model to simulate and predict the spreading of the newly discovered disease caused by a new series of a Novel Coronavirus (COVID-19). The mathematical modeling in this study is based on the Susceptible Infected Recovery (SIR) model, where key controlling parameters are considered, namely: human contact factor b, transmit factor (a), health medication factor (m) and initial infected (I0). The simulation results show the effect of these parameters, and their role in spreading the COVID-19. The results also show that by keeping a high medication factor and a low contact factor, the spreading of COVID-19 will slow down. The medication health factor depends on the infrastructure of a country, and it is difficult to improve it instantly. On the other hand, the contact factor can be easily controlled. Enforcing the physical social distancing, drastically decreases the contact factor. Hence, slow down the spreading of the virus. Also, the effect of medication factor on the number deaths caused by COVID-19 is studied. The results show that as medication factor increases the number of deaths decreases.

1. Introduction

As of April 22, 2020, almost the whole world is on lockdown due to the significant spread of a highly contagious corona virus (COVID-19) everywhere [1–5]. According to the Center for Systems Science and Engineering (CSSE) at Johns Hopkins [6], the COVID-19 has resulted in 2626,929 confirmed cases and still counting. 710,285 cases have recovered while, unfortunately, 183,283 people have died. The most dangerous feature of COVID-19 is its ability to spread quickly. In only three months, since it was first discovered in the city of Wuhan, Hubei Province, China in December 2019, the virus has reached into every corner of the world. The number of infected people is continuously increasing at an unprecedented infection rate. The contagious nature of the COVID-19 is both new and complicated [7]. The unknown part is that this virus has up to 14 days incubation time, where the carrier may not show any symptoms. Moreover, some infected people may not show any symptoms "asymptomatic" altogether. In addition, the health authorities have not yet certainly traced the origin of COVID-19.

The COVID-19 has forced countries worldwide to close their borders, shutdown businesses, keep people inside their homes, and many other prevention measures in order to slow down its spreading. Fig. 2 shows the number if COVID-19 infected cases in Jordan [8]. Jordan is one of few countries that managed to keep the COVID-19 case relatively very low by apply a strict social distancing and a curfew as early as first case was discovered in the country. This experience offers a solid indication for the role of social contact in spreading the disease. However, this caused radical effects on the day-to-day life as well as national and international economies. Many

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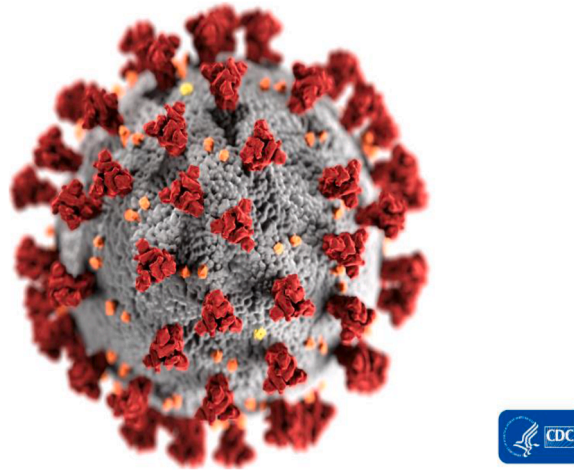
Fig. 1. A depiction of the Novel Coronavirus (COVD-19). (Source: https://tvhcare.org/tvh-press-conference-03-14-2020/).
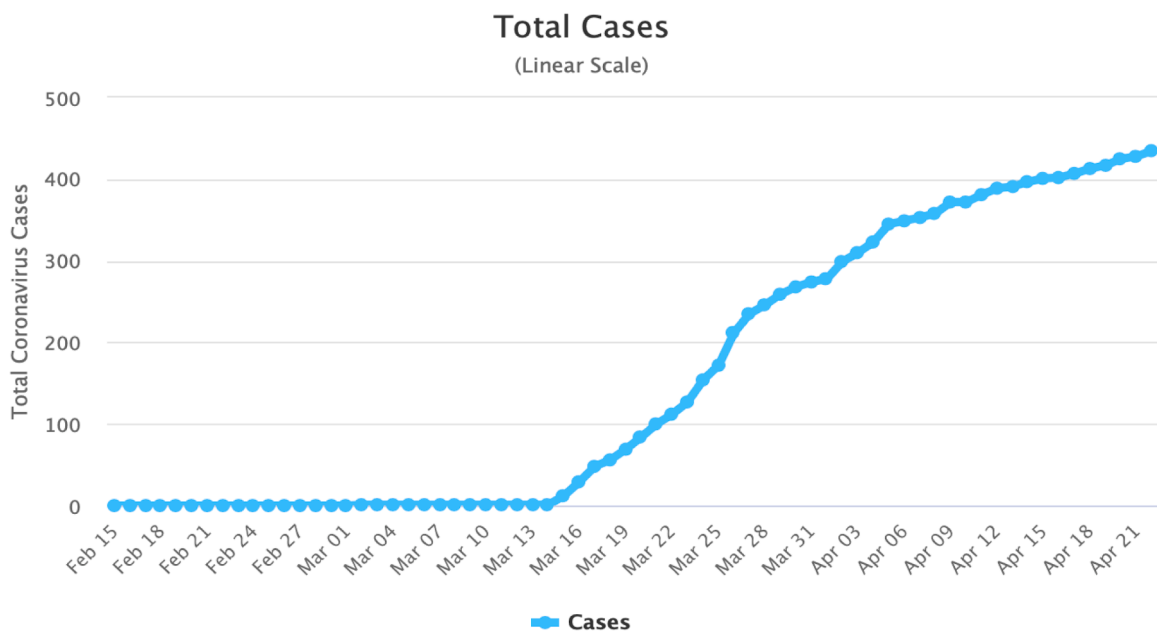

Fig. 2. COVID-19 Infected cases in Jordan.

people have lost their jobs, large corporations have started shutting down research projects to save on expenses, and many local small businesses were completely closed. According to the US Bureau of labor statistics [9], the number of unemployed persons is 23.1 million in April, with 14.7% unemployment rate. This urgently forces all governments to face a very challenging question: when will this end?

The SIR model was first proposed to study the spread of the "Hong Kong Flu" [9]. Recently, many publications have adopted this model or a modified version [7,10,11,12]. Usually, most models only focus on the human-to-human transmission of the diseases. However, the model proposed in [7] considers different ways for the disease to be transmitted with an environmental-based transmission rate.

In this paper, four key controlling parameters are considered in the SIR (or SIRD) model, and their role in spreading or containing the COVID-19. The number of infected person contacts daily, and the quality of the health system have a critical role in stabilizing, and even eliminating the spread of the disease. These parameters can be controlled on individual as well as governmental level, which helps the authorities in setting new controlling measures and plans for any similar situation in the future.
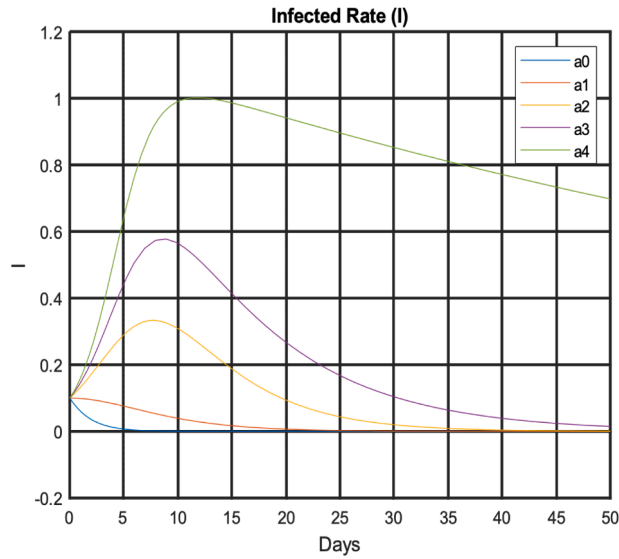
Fig. 3. The relation of I vs time, where a0 > a1>a2 > a3 >a4.

2. Mathematical nonlinear COVID-19 model

The mathematical COVID-19 model is a modified version of the general SIR model [10]. The proposed model is a fourth order nonlinear differential equation that describes the most crucial parameters of a contiguous disease, such as **S**usceptible (**S**), **I**nfected (**I**), **R**ecovery (**R**), and **D**eath (**D**). The state space fourth order nonlinear mathematical model **SIRD** is presented next.

2.1. SIRD nonlinear model

$$
\begin{aligned}
\frac{dS}{dt} &= -bSI \\
\frac{dI}{dt} &= bSI - aI \\
\frac{dR}{dt} &= aI \\
\frac{dD}{dt} &= -(bSI - 2aI) - m * D
\end{aligned}
\tag{1.1}
$$

Where: b is defined as the contact factor
a is defined as the transmit factor
m is defined as the health medication quality factor
Let $x_1 = S, x_2 = I, x_3 = R,$ and $x_4 = D,$
Then, SIRD model can be described in sate space model as follows:

$$
\begin{aligned}
\frac{dx_1}{dt} &= -bx_1x_2 \\
\frac{dx_2}{dt} &= bx_1x_2 - ax_2 \\
\frac{dx_3}{dt} &= ax_2 \\
\frac{dx_4}{dt} &= -(bx_1x_2 - 2ax_2) - m * x(4)
\end{aligned}
\tag{1.2}
$$

In general,

$$
\dot{X} = f(\mu, X)
\tag{1.3}
$$

Where Eq. (1.3) is a nonlinear differential equation, and μ is a control parameter vector, such as *b, a,* and *m.*
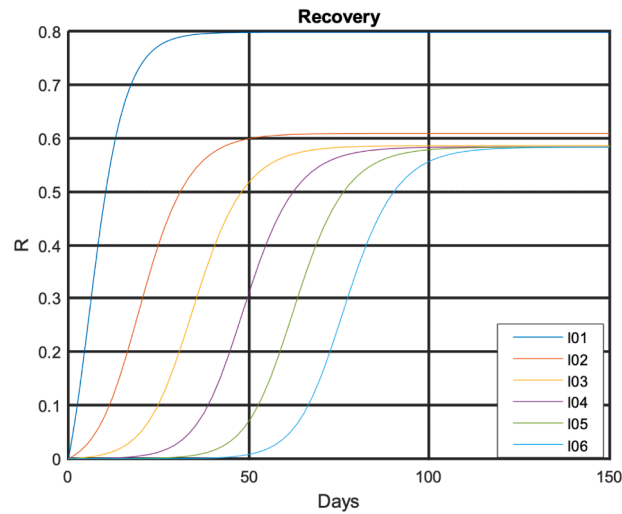
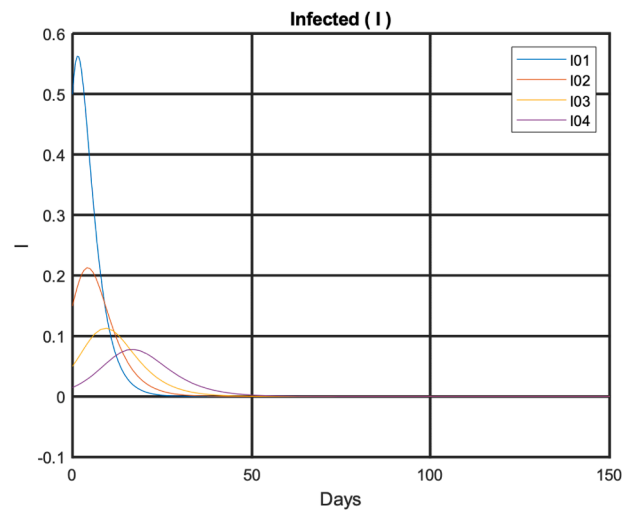**Fig. 4.** Relation of *R* vs Time, where I01<I02<I03<I04<I05<I06.

Fig. 5. Relation of *I* vs Time, where: *I01 < I02 < I03 < I04*.

2.2. Initial conditions (operating point)

The initial condition can be found by equating Eq.(1.3) to zero, and solve for ***X(0)***. The following variables are defined at the beginning of the spread of the disease.

$S(0)$: number of population in any country
$I(0)$: number of infected people
$R(0)$: number of recovered people, $R(0)=0$
$D(0)$: number of death at the beginning, $D(0)=0$.

We define a new parameter, R0 as a function of b and a, where $R0 = \frac{b}{a}$. Based on the operating point, we found when $R0 < 1$, means that No corona pandemic, but when $R0 > 1$, the corona pandemic starts to spread out.

3. Simulation results and discussion

In this section, the time history simulation of main four variables S, I, R, and D is investigated. **Fig. 3** shows the infected parameter (I), starting from $I(0) = 0.1$, changing with time based on one control parameter "transmit parameter (*a*)". The results show that as this parameter (*a*) decreases the infected numbers decreases.

Fig. 4 shows the effect of changing the infected value (*I*) on the recovery parameter (*R*). The study shows that with a small value of *I0, R* increases to higher value in a very short time. While, as I0 increases, the final value of R decreases.
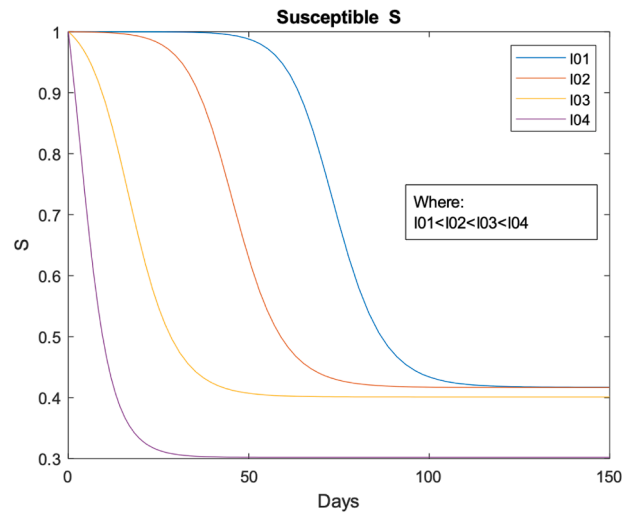
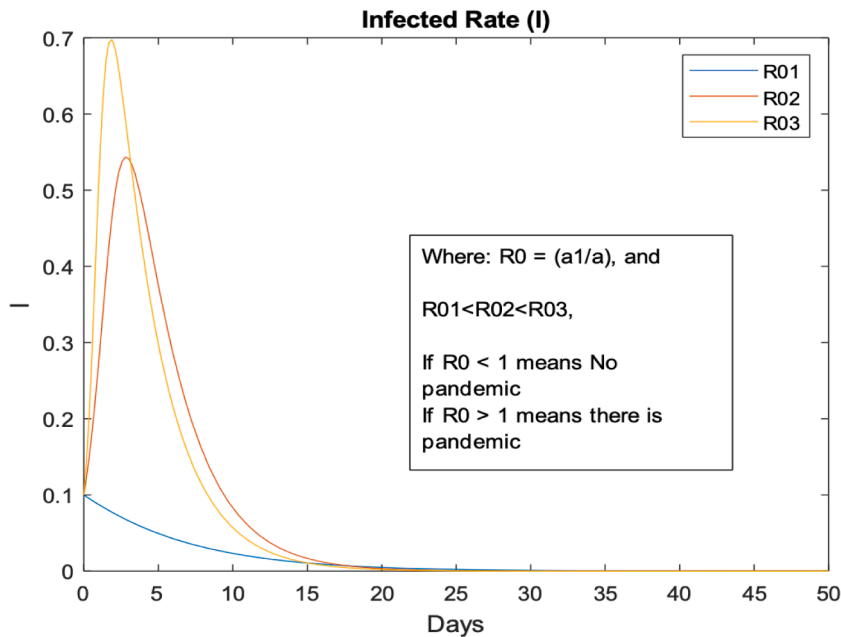Fig. 6. Time history of Susceptible (S) with changing the initial infected values.

Fig. 7. The infected parameter (I) time history with different values of.

The effect of the initial number of infected (**I0**) on the infected rate versus time is shown in **Fig. 5**. As it can be seen below, the I0 has a profound effect on the infected rate. As I0 increases, I decreases. with a large I0, there are smaller number (out of S) to get infected. On the other hand, a smaller I0 results in a higher peak in I. However, it will sharply decay down.

In **Fig. 6**, the susceptible (**S**) time history is shown for different values of the infected (**I**). The study shows that as the initial value of infected people (**I0**) increases, the susceptible (**S**) decreases very quickly to a lower value than in the case of smaller **I0**.

Next, a new parameter is introduced in this study, based on the operating point of the dynamical system given in Eq. (1), named R0. Where R0 $= \frac{b}{a}$. If this value for R0 is less than unity, means no pandemic occur, but if R0 greater than unity, means the virus will spread out resulting in a pandemic as shown in **Fig. 7**, where R01 < 1, while R02 and R03 > 1.

Finally, one crucial variable of the SIRD mathematical model is the death variable (D). It is analyzed in **Fig. 8**, considering the new parameter introduced as "**medication health quality**" (**m**). The model shows that as the medication quality increases the number of potential deaths decreases. Starting with zero medication quality factor (**m1 = 0**), as expected the number of deaths increase rapidly. Then, m is increased, and the number of deaths decrease in a very effective way. As it reaches a 100% medication quality factor (m5 =1), the minimum number of deaths can be achieved.
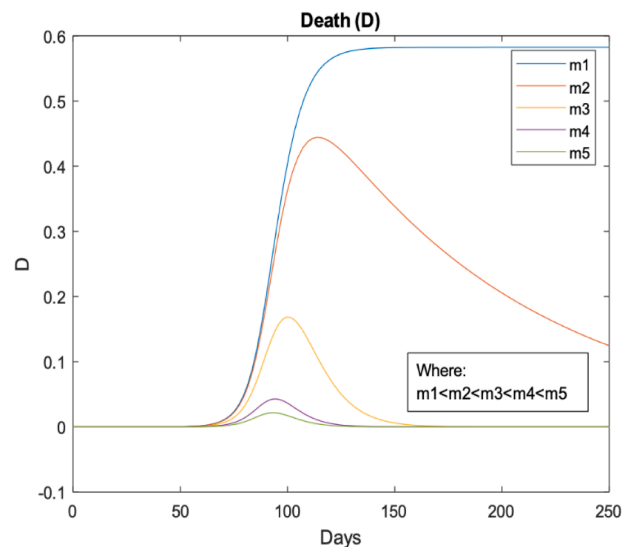
Fig. 8. Death vs Time, with changing the medication quality factor *m*.

Conclusion

A modified SIRD model was used to simulate and model the spreading of the COVID-19 virus. The model studied critical controlling parameters, and their effect on spreading the virus, such as *b, a,* and *m.* The nonlinear behavior is obvious in all cases, where the number of infected peaks up and come down exponentially. The rate of increase and decrease are affected by the studied controlling parameters. A new control parameter R0, which depends on (*a* and *b*) is studied. The study showed when R0 ⟨ 1, no pandemic will happen, but when R0 ⟩ 1, the virus will spread causing a pandemic. Finally, the study showed that as the medication quality factor decreases, the number of deaths will increase rapidly, while with high medication quality factor the death number of people will decrease rapidly.

Author Statement

Dear Editor in Chief: Hope this message finds you well. I am submitting the revised version of my paper which has number COMPELECENG-D-20-00415, and the title: Corona COVID-19 Spread- a Nonlinear Modeling and Simulation

Declaration of Competing Interest

None.

References

[1] P. Zhou, X.L. Yang, X.G. Wang, B. Hu, L. Zhang, W. Zhang, et al., Discovery of a novel coronavirus associated with the recent pneumonia outbreak in humans and its potential bat origin, bioRxiv, 2020.
[2] Gralinski LE, Menachery VD. Return of the coronavirus: 2019-nCoV. Viruses 2020;12:135.
[3] Centers for Disease Control and Prevention: 2019 novel coronavirus. Available from: https://www.cdc.gov/coronavirus/2019-ncov, 2019.
[4] Li Q, Guan X, Wu P, Wang X, Zhou L, Tong Y, et al. Early transmission dynamics in Wuhan, China, of novel coronavirus-infected pneumonia. N Engl J Med 2020.
[5] World Health Organization, "Novel coronavirus - China." https://www.who.int/csr/don/12-january-2020-novel-coronavirus-china/en/, 2020. [Online; accessed 17-January-2020].
[6] Coronavirus COVID-19 Global Cases by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins (COVID-19 Map - Johns Hopkins Coronavirus Resource Center), 2020.
[7] Chayu Yang and Jin Wang, "A mathematical model for the novel coronavirus epidemic in Wuhan, China" - https://www.worldometers.info/coronavirus/country/jordan/, 2020.
[8] US Bureau of labor statistics, "Unemployment rate rises to record high 14.7 percent in April 2020".
[9] Smith David, Moore Lang. The SIR model for spread of disease - introduction. Convergence December 2004.
[10] Wu J-T, Leung K, Leung GM. Nowcasting and forecasting the potential domestic and international spread of the 2019-nCoV outbreak originating in Wuhan, China: a modeling study. Lancet 2020;395:689–97.
[11] Victor Alexander Okhuese, "Mathematical prediction for COVID-19 as a global pandemic", 2020.
[12] Ming WK, Huang J, Zhang CJ. Breaking down of healthcare system: mathematical modelling for controlling the novel coronavirus (2019-nCoV) outbreak in Wuhan, China. bioRxiv 2020.

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